Volume Rendering in the Gradient Domain

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$I_j = \int_{\Omega} f_j(\bar{x}) d\mu(\bar{x})$

[Pauly et al. 00]
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\[ f_j(\bar{x}) = L_e(x_0 \rightarrow x_1) \]
\[ \times G_v(x_0, x_1) \times T(x_0, x_1) \]
\[ \times f_s(x_0 \rightarrow x_1 \rightarrow x_2) \]
\[ \times G_v(x_1, x_2) \times T(x_1, x_2) \]
\[ \times W_j(x_1 \rightarrow x_2) \]
VOLUMETRIC MEASUREMENT CONTRIBUTION FUNCTION

- Surface
  - Transport between two points is $G$
  - Light scatters on surface by $f_r$
- Volume
  - Transport between two points is $G_v T$
  - Light scatters in volume by $f_s$
PATH SAMPLING IN VOLUMES
\[ I_j = \int_{\Omega} f_j(\bar{x}) d\mu(\bar{x}) \]
\[ I_j \approx \frac{1}{N} \sum_{i=1}^{N} \frac{f_j(\bar{x}_i)}{p(\bar{x}_i)} \]
\[ p(\bar{x}_i) = p(x_0) \times p(x_1|x_0) \times p(x_2|x_1) \]
SAMPLING PATHS - RAY TRACING

\[ p(\bar{x}_i) = p(x_0) \]
SAMPLING PATHS - RAY TRACING

\[ p(\bar{x}_i) = p(x_0) \times p_\omega(x_0 \rightarrow x_1) \]
\[ p(\bar{x}_i) = p(x_0) \times p_\omega(x_0 \rightarrow x_1) \times p_t(|x_0 - x_1|) \]

Distance sampling
$p(\bar{x}_i) = p(x_0)$

$\times p_\omega(x_0 \rightarrow x_1)$

$\times p_t(|x_0 - x_1|)$

$\times G_v(x_0, x_1)$
\[ p(\bar{x}_i) = p(x_0) \times p_\omega(x_0 \rightarrow x_1) \times p_t(|x_0 - x_1|) \times G_v(x_0, x_1) \times p_\omega(x_1 \rightarrow x_2) \times p_t(|x_1 - x_2|) \times G_v(x_1, x_2) \]
\[
p(\bar{x}_i) = p(x_0) \times p_\omega(x_0 \rightarrow x_1) \times p_t(|x_0 - x_1|) \times G_v(x_0, x_1) \times p_\omega(x_1 \rightarrow x_2) \times p_t(|x_1 - x_2|) \times G_v(x_1, x_2)
\]
\[ p(\bar{x}_i) = p(x_0) \]
\[ \times p(x_1 | x_0) \]
\[ \times p_\omega(x_1 \rightarrow x_2) \]
\[ \times p_t(|x_1 - x_2|) \]
\[ \times G_v(x_1, x_2) \]
p(\bar{x}_i) = p(x_0) \times p(x_1 | x_0) \times p(x_2 | x_1)
PATH SAMPLING IN VOLUMES

- Surface
  - Sample a point on a surface
  - Distance to the next point is given
- Volume
  - Sample a point in a volume
  - Distance to the next point is sampled
RENDERING ALGORITHMS
VOLUMETRIC PATH TRACING

From eye

[Lafortune and Willems 96]
VOLUMETRIC BIDIRECTIONAL PATH TRACING

From eye
From light

[Lafortune and Willems 96]
VOLUMETRIC PHOTON DENSITY ESTIMATION

From light

From eye

[Jensen and Christensen 98]

[Hachisuka et al. 17]
VOLUMETRIC PHOTON DENSITY ESTIMATION

From eye
From light

[Jensen and Christensen 98]
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VOLUMETRIC PHOTON DENSITY ESTIMATION

From light
From eye

[Jensen and Christensen 98]
[Hachisuka et al. 17]
VOLUMETRIC PHOTON DENSITY ESTIMATION

From eye
From light

Use a 3D blur kernel to relate disconnected path segments

[Jensen and Christensen 98]
[Hachisuka et al. 17]
BEAM RADIANCE ESTIMATE

From light

From eye

[Jarosz et al. 08]
[Hachisuka et al. 17]
BEAM RADIANCE ESTIMATE

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[Hachisuka et al. 17]

[Jarosz et al. 08]

[Hachisuka et al. 17]
BEAM RADIANCE ESTIMATE

From light
From eye

Use a 2D blur kernel at the projected point

[Hachisuka et al. 17]
[Jarosz et al. 08]
PHOTON BEAMS

From light
From eye

[Hachisuka et al. 17]
[Jarosz et al. 11]
PHOTON BEAMS

From light

From eye

[Hachisuka et al. 17]

[Jarosz et al. 11]
PHOTON BEAMS

From light
From eye

[Hachisuka et al. 17]
[Jarosz et al. 11]
PHOTON BEAMS

From eye
From light

Use a 1D blur kernel between closest points

[Hachisuka et al. 17]
Jarosz et al. 11]
PHOTON PLANES

From eye
From light

[Bitterli and Jarosz 17]
PHOTON PLANE$\text{s}$

From eye
From light

[Bitterli and Jarosz 17]
PHOTON PLANES

From eye
From light

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PHOTON PLANES

From eye
From light

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PHOTON PLANES

From eye
From light

[Bitterli and Jarosz 17]
PHOTON PLANES

Deterministic projection to the intersection point (termed “0D” blur)

From eye
From light

[Bitterli and Jarosz 17]
FOUR KEY POINTS

• Volumetric light transport is still an integration
• Vertices can be anywhere in 3D space
• Distance sampling is necessary
• Density estimation has a lot more variation
APPENDIX
VOLUMETRIC MEASUREMENT CONTRIBUTION FUNCTION

\[ f_j(\bar{x}) \]
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\[ \bar{x} = x_0 x_1 x_2 \]
VOLUMETRIC MEASUREMENT CONTRIBUTION FUNCTION

$$f_j(\bar{x}) = L_e(x_0 \rightarrow x_1)$$

Emission from a light source
VOLUMETRIC MEASUREMENT CONTRIBUTION FUNCTION

\[ f_j(\bar{x}) = L_e(x_0 \rightarrow x_1) \times G(x_0, x_1) \]

Geometric throughput
VOLUMETRIC MEASUREMENT CONTRIBUTION FUNCTION

\[ f_j(\bar{x}) = L_e(x_0 \rightarrow x_1) \times G_v(x_0, x_1) \]

Geometric throughput
VOLUMETRIC MEASUREMENT CONTRIBUTION FUNCTION

\[ f_j(\bar{x}) = L_e(x_0 \rightarrow x_1) \times G_v(x_0, x_1) \times T(x_0, x_1) \]

Attenuation due to volume
VOLUMETRIC MEASUREMENT CONTRIBUTION FUNCTION

\[ f_j(\bar{x}) = L_e(x_0 \rightarrow x_1) \]
\[ \times G_v(x_0, x_1) \times T(x_0, x_1) \]

Volumetric throughput between two points
VOLUMETRIC MEASUREMENT CONTRIBUTION FUNCTION

\[ f_j(\bar{x}) = L_e(x_0 \rightarrow x_1) \]
\[ \times G_v(x_0, x_1) \times T(x_0, x_1) \]
\[ \times f_s(x_0 \rightarrow x_1 \rightarrow x_2) \]

Scattering from \( x_0 \) to \( x_1 \) to \( x_2 \)
VOLUMETRIC MEASUREMENT CONTRIBUTION FUNCTION

\[ f_{j}(\bar{x}) = L_{e}(x_0 \rightarrow x_1) \times G_{v}(x_0, x_1) \times T(x_0, x_1) \times f_{s}(x_0 \rightarrow x_1 \rightarrow x_2) \]

Scattering from \( x_0 \) to \( x_1 \) to \( x_2 \)

\[ f_{s}(x_0 \rightarrow x_1 \rightarrow x_2) = \sigma_{s}(x_1) \rho(x_0 \rightarrow x_1 \rightarrow x_2) \]
VOLUMETRIC MEASUREMENT CONTRIBUTION FUNCTION

\[ f_j(\bar{x}) = L_e(x_0 \rightarrow x_1) \times G_v(x_0, x_1) \times T(x_0, x_1) \times f_s(x_0 \rightarrow x_1 \rightarrow x_2) \times G_v(x_1, x_2) \times T(x_1, x_2) \]

Volumetric throughput between two points
VOLUMETRIC MEASUREMENT CONTRIBUTION FUNCTION

\[ f_j(\bar{x}) = L_e(x_0 \rightarrow x_1) \times G_v(x_0, x_1) \times T(x_0, x_1) \times f_s(x_0 \rightarrow x_1 \rightarrow x_2) \times G_v(x_1, x_2) \times T(x_1, x_2) \times W_j(x_1 \rightarrow x_2) \]

Sensor response of the pixel j